AD-A034 433 NAVAL AIR DEVELOPMENT CENTER WARMINSTER PA SYSTEMS DEPT F/G 17/1
MOTION CONSTRAINTS REQUIRED OF A MONITORING AIRCRAFT FOR UNAMBI--ETC(U)
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MOTION CONSTRAINTS REQUIRED OF A MONITORING AIRCRAFT FOR UNAMBIGUOUS BEARING ONLY TRACKING

Dr. I. R. Goodman
Systems Department
NAVAL AIR DEVELOPMENT CENTER
Warminster, Pennsylvania 18974

15 NOVEMBER 1976

FINAL REPORT
AIRTASK NO. A5335330/0012/7240000000

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

Prepared for
NAVAL AIR SYSTEMS COMMAND
Department of the Navy
Washington, D.C. 20361

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 14 NADC-76379-50

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle) 6 Motion Constraints Required of a Monitoring Aircraft for Unambiguous Bearing Only Tracking.

5. TYPE OF REPORT & PERIOD COVERED 9 Final Report,

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s) 10 Dr. I. R. Goodman

8. CONTRACT OR GRANT NUMBER(s) 12 24p.

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Naval Air Development Center
Systems Department (551)
Warminster, PA 18974

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
AIRTASK NO.: A5335330/0012/7240000000

11. CONTROLLING OFFICE NAME AND ADDRESS
Naval Air Systems Command
Department of the Navy
Washington, DC 20361

12. REPORT DATE 11 15 NOV 1976

13. NUMBER OF PAGES 21

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report) UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Bearing Only Tracking
Unambiguous Tracking

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes the equations of motion required of a monitoring aircraft to track successfully a moving target emitter by sequential bearing only information. When the monitoring aircraft and the target both describe straight line constant velocity motion, unambiguous tracking cannot be carried out, unless a priori knowledge of at least one of the target's parameters of motion is available. However, this ambiguity is resolvable for most real situations, when the monitoring aircraft

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S/N 0102-LF-014-6601

403 731 UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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20. ABSTRACT (cont'd)

purposely describes nonlinear motion and the target is still assumed to have linear motion. If the former's motion is of a sufficiently high degree of nonlinearity, such as circular motion, no ambiguities whatsoever will occur in any tracking situation.

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SUMMARY

The basic sequential bearing only tracking problem for targets and sonobuoys by a moving monitoring aircraft is examined here for uniqueness of solutions for a given bearing observation set. When the monitoring aircraft describes only linear motion, an infinite class of ambiguous solutions arises, the ambiguity only being resolved if prior knowledge of at least one of the parameters is obtainable. However, if the monitoring aircraft purposely describes non-linear motion, in general, except for certain linearly restricted sets of target motion parameter values, which may be assumed to occur in practice with probability zero, no prior knowledge of the target's parameters of motion is required to obtain a unique solution for tracking from a given bearing observation set. If the motion of the monitoring aircraft is, in particular, circular, or of a sufficiently high degree of non linearity, an unambiguous solution is always obtained with no restrictions whatsoever on the target motion parameters.

INTRODUCTION

Sensors on the P-3C aircraft, sequentially in time, measure bearing lines between the aircraft and a moving target emitter. The objective of such measurements is to determine the emitter position and velocity, assuming for simplicity, straight line constant velocity motion for the target or sonobuoy.

This report discusses the type of motion required of the P-3C aircraft in order to track successfully. (A somewhat analagous problem arises with the use of a single DIFAR sonobuoy. The sonobuoy is stationary and linear motion is assumed for the target.)

CONCLUSIONS

When sequential bearings are measured between the P-3C aircraft and emitter, a non-linear (second degree or higher polynomial equation of motion) flight path, must be flown by the monitoring aircraft to track the position (and velocity) the the emitter.

If only a linear flight path is flown by the P-3C aircraft, than a known target velocity is required for tracking, in order to obtain an unambiguous solution.

STATEMENT OF PROBLEM

Bearing only sensors emanating from a monitoring (P-3C) aircraft are used to track a target, assumed to travel in straight line constant velocity motion. Given several observed (generally, in error) bearings between the aircraft and target, is enough information present to track?

ANALYSIS FOR LINEAR MONITORING AIRCRAFT MOTION AND LINEAR TARGET MOTION

We show that, in general, there is no unambiguous solution (even for

the zero error tracking case). From this it follows that no correct regression solution can exist for tracking with bearing errors present for the friendly aircraft. However, if a priori one of the parameters of motion of the target is known, the ambiguity can be resolved.

Consider first the following figure illustrating the problem for the zero error case:

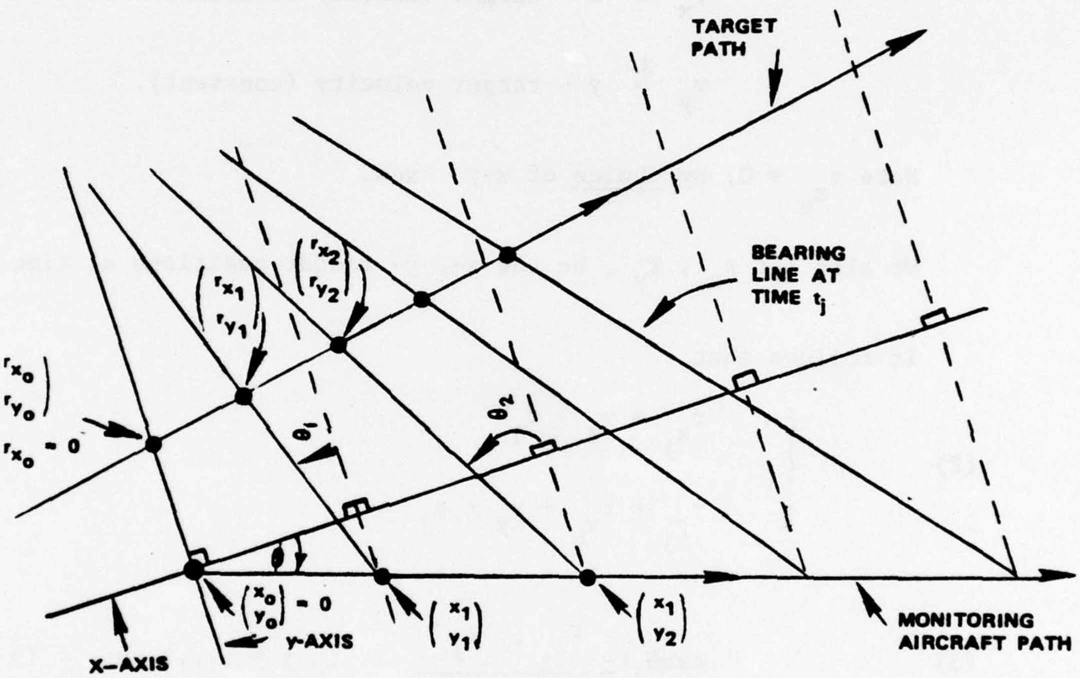


FIGURE 1.

We assume for j = 0, 1, 2,...:

Knowns: $t_j \overset{df}{=}$ time of j^{th} bearing fix; $t_0 \overset{df}{=} 0$,

$s \overset{df}{=}$ speed of monitoring aircraft,

$\phi \overset{df}{=}$ angle between x-axis and monitoring aircraft path,

$\theta_j \overset{df}{=}$ angle between j^{th} successive bearing line and y-axis,

(1) $$x_j = s \cdot \cos(\phi) \cdot t_j$$
$$y_j = s \cdot \sin(\phi) \cdot t_j,$$

x-y positions of monitoring aircraft at time t_j

<u>Unknowns</u>: $r_{y_o} \overset{df}{=}$ y - initial (t_o) target position

$v_x \overset{df}{=}$ x - target velocity (constant)

$v_y \overset{df}{=}$ y - target velocity (constant).

Note $r_{x_o} = 0$, by <u>choice</u> of x-y xes.

We also let r_x, r_y, be the x-, y- target positions at time t_j.

It follows that

(2) $$\begin{cases} r_{x_j} = v_x \cdot t_j \\ r_{y_j} = r_{y_o} + v_y \cdot t_j \end{cases}$$

(3) $$\tan(\theta_j) = \frac{r_{x_j} - x_j}{r_{y_j} - y_j} \quad ; \; j = 1, \ldots, n \qquad (n \geq 2)$$

Using (1) and (2) in (3) yields immediately

(4) $$\tan(\theta_j) = \frac{U \cdot t_j}{V + W \cdot t_j} \quad ; j = 1,.,n$$

where

$$U \overset{df}{=} v_x - s \cdot \cos(\phi)$$
$$V \overset{df}{=} r_{y_o}$$
$$W \overset{df}{=} v_y - s \cdot \sin(\phi)$$

are unknown, since r_{y_o}, v_x, v_y are unknown.

Clearly, if (U, V, W) satisfies the equations so does $(\lambda\cdot U, \lambda\cdot V, \lambda\cdot W)$, for any real $\lambda \neq 0$.

Thus, dividing the right hand side of (4) through by $U \neq 0$, we obtain with a little algebra

(5) $$t_j \cot\theta_j = \frac{V}{U} + \frac{W}{U} \cdot t_j \; ; \; j = 1,..,n \; .$$

Hence we have the matrix relation:

(6) $$\underbrace{\begin{pmatrix} t_1 \cot(\theta_1) \\ \vdots \\ t_n \cot(\theta_n) \end{pmatrix}}_{=Z} = \underbrace{\begin{pmatrix} 1 & t_1 \\ \vdots & \vdots \\ 1 & t_n \end{pmatrix}}_{=B} \cdot \underbrace{\begin{pmatrix} V/U \\ W/U \end{pmatrix}}_{=X} \qquad (n \geq 2)$$

for the zero error situation. Since trivially $0 = t_0 < t_1 < .. < t_n$, B is n by2 of full rank 2, and thus either (6) has no solution or a unique solution for V/U, W/U. (If X and X' are two solutions, then $BX = BX'$ implies $0 = B \cdot (X-X')$ and premultiplying by $(B^T B)^{-1} \cdot B^T$ yields at once $X-X'=0$.)

Because the zero error "observations" θ_j, and hence $t_j \cdot \cot(\theta_j)$, $j=1,...,n$, are generated by a physically realizable target, corresponding to some U, V, W, and hence X, (6) will have a unique solution for X when a target describing straight line motion is present.

The solution X can be written

(7) $$X = \begin{pmatrix} 1 & t_1 \\ 1 & t_2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} t_1 \cdot \cot(\theta_1) \\ t_2 \cdot \cot(\theta_2) \end{pmatrix} \text{, or ,}$$

since we have perfect information,

$$X = (B^TB)^{-1} \cdot B^T z, \tag{8}$$

in a more redundant form.

However, for any such X, there are <u>infinitely many triples (U, V, W) that give this ratio</u>, since we have trivially U arbitrary, $V = U \cdot \frac{V}{U}$, $W = U \cdot \frac{W}{U}$, although $\frac{V}{U}$ and $\frac{W}{U}$ are unique.

Thus, for even the zero error case, the solution set of (4) is:

(9) U arbitrary, $V = U \cdot A$, $W = U \cdot B$,
where $X = \begin{pmatrix} A \\ B \end{pmatrix}$ is the unique solution of (6) given in (7) or (8).

More generally, if θ_i is observed in error as $\overset{\circ}{\theta}_i$ and if we assume $\overset{\circ}{z}_i \overset{df}{=} t_i \cot \overset{\circ}{\theta}_i$, $z_i \overset{df}{=} t_i \cot \theta_i$, $\overset{\circ}{z} \overset{df}{=} \begin{pmatrix} \overset{\circ}{z}_1 \\ \vdots \\ \overset{\circ}{z}_n \end{pmatrix}$, $z \overset{df}{=} \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix}$,

$E(\overset{\circ}{z} \mid X) = z$, and $Cov(\overset{\circ}{z} \mid X) = \Sigma$, a known positive definite matrix, not dependent on X, for all X, then (4) is replaced by

$$\tan(\theta_j) = \frac{U \cdot t_j}{V + W \cdot t_j} + \text{error}_j \;; \quad j = 1, \ldots, n, \tag{10}$$

and hence (5) becomes

$$t_j \cdot \cot(\overset{\circ}{\theta}_j) = \frac{V}{U} + \frac{W}{U} t_j + \text{error}_j \;; \quad j = 1, \ldots, n, \tag{11}$$

and analagous to (6) we have the linear regression model

$$\overset{\circ}{z} = B \cdot X + \epsilon, \text{ where} \tag{12}$$

$$\epsilon \overset{df}{=} \overset{\circ}{z} - z = \text{error},$$

$$E(\epsilon) = 0, \quad Cov(\epsilon) = \Sigma,$$

Hence, the weighted least square, etc. estimator of X is

(13) (a) $$\hat{X}(\overset{o}{Z}) = (B^T \cdot \Sigma^{-1} \cdot B)^{-1} \cdot B^T \; \Sigma^{-1} \overset{o}{Z}$$

(b) $$E(\hat{X}(\overset{o}{Z}) \mid X) = X,$$

(c) $$\mathrm{Cov}\;(\hat{X}(\overset{o}{Z}) \mid X) = (B^T \cdot \Sigma^{-1} \cdot B)^{-1}.$$

(See e.g., reference 1, Chapter 4, for complete details.)

Thus, analogous to (9), the set of 'optimal' estimators $(\hat{U}, \hat{V}, \hat{W})$ of (U, V, W) in (10) is given by:

(14) $$\hat{U} \text{ arbitrary}, \quad \hat{V} = \hat{U} \cdot \hat{A}, \quad \hat{W} = \hat{U} \cdot \hat{B}, \text{ where}$$

$$\hat{X} = \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} \text{ is given in (13a)}$$

Again, we state: <u>there is an infinite number of triples</u> $(\hat{U}, \hat{V}, \hat{W})$ leading to the (unique) ratios $(\widehat{V/U})$, $(\widehat{W/U})$.

Thus, for either the zero error case in (4) or more generally the error case in (10), the solution set of (U, V, W)'s is infinite, and unresolvable ambiguity for all solutions holds.

However, if <u>one</u> of the parameters U, V, or W is known a priori, then the ambiguity is completely resolved: Since if V or W is known, a non-linear relation will always hold in (4) or (10) between any function of θ_j and the unknown parameters, we consider without loss of generality, and for simplicity, the situation when U is known:

In this case, (12) becomes:

$$(15) \qquad \mathring{z}' = B \cdot X' + \epsilon',$$

where $\mathring{z}' \stackrel{df}{=} \begin{pmatrix} \mathring{z}_1' \\ \vdots \\ \mathring{z}_n' \end{pmatrix}$, $\mathring{z}_j' \stackrel{df}{=} U \cdot t_j \cot(\mathring{\theta}_j)$; $j = 1, \ldots, n$,

and $X' \stackrel{df}{=} \begin{pmatrix} V \\ W \end{pmatrix}$; $E(\epsilon') = 0$, $Cov(\epsilon') = \Sigma'$, known,

$$\hat{X}' = (B^T \cdot \Sigma'^{-1} \cdot B)^{-1} \cdot B^T \Sigma'^{-1} \mathring{z}'$$

$$= \begin{pmatrix} \hat{V} \\ \hat{W} \end{pmatrix}, \text{ etc.}$$

No ratios appear and $\hat{X}'$ is unique.

Thus, the optimal estimator of (U, V, W) in (15) is

$$(16) \qquad (U, \hat{V}, \hat{W}), \text{ uniquely.}$$

A similar unique solution for U, V, W holds for the zero error case when U is known.

ANALYSIS FOR NONLINEAR MONITORING AIRCRAFT MOTION AND LINEAR TARGET MOTION

We show that in general when the monitoring aircraft purposely describes nonlinear motion, the unknown target generating the observed bearing lines is essentially uniquely determined by solving the bearing line equations. Any ambiguity in the general case only occurs (in general with probability zero, given all possible admissible values equally probable, a priori) when certain linear restrictions hold for the unknown parameters of target motion.

Knowns: $t_j \stackrel{df}{=}$ time of j^{th} bearing fix;

$$(17) \qquad x_j \stackrel{df}{=} \alpha_1 t_j + \alpha_2 t_j^2 + \alpha_3 t_j^3 + a \cdot O_x(t_j^4)$$

$$y_j \stackrel{df}{=} \beta_1 t_j + \beta_2 t_j^2 + b \cdot O_y(t_j^3) \quad ,$$

$$j = 0, 1, 2, \ldots$$

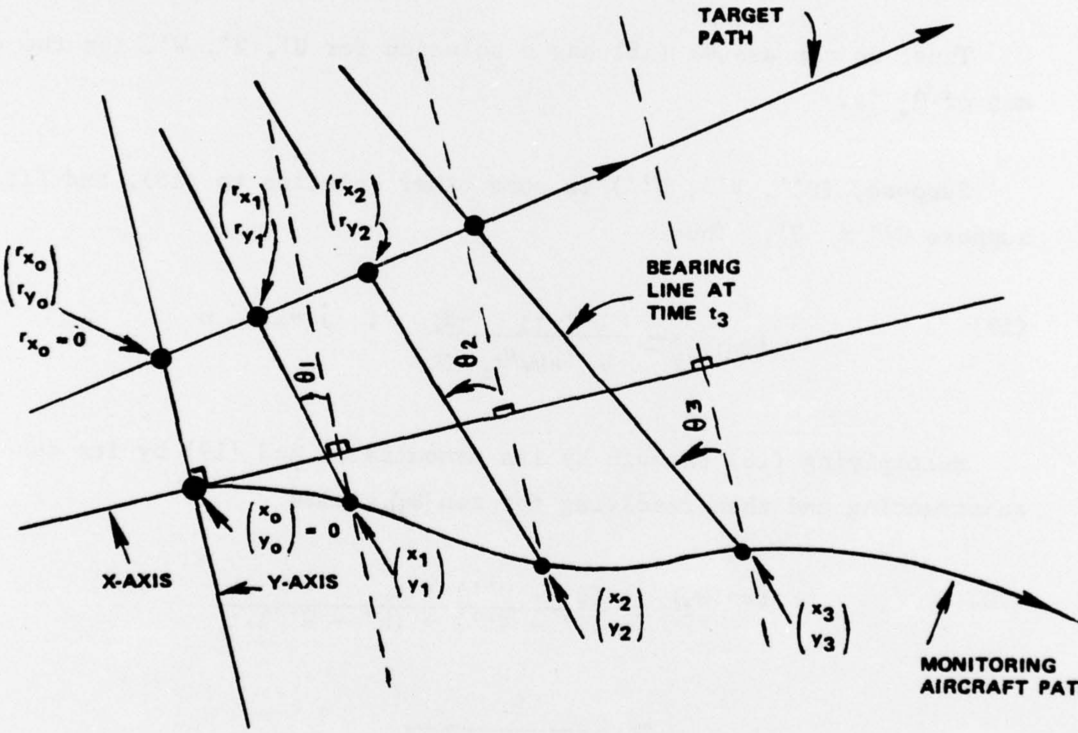
x-y positions of friendly aircraft at time t_j, $\alpha 1$, $\alpha 2$, $\alpha 3$, $\beta 1$, $\beta 2$, a, b all known. α_3, a, or b may be zero, but α_2 and $\beta 2 \neq 0$.

<u>Unknowns</u>: $r_{y_o} \overset{df}{=}$ y - initial (to) target position

$V_x \overset{df}{=}$ x - target velocity (constant)

$V_y \overset{df}{=}$ y - target velocity (constant)

Again, $r_{x_o} = 0$, by choice of x-y axes. (See Figure 2 below.)



<u>FIGURE 2.</u>

Equations (2) and (3), with the new interpretation remain valid here.

We first consider the zero error case.

Using (17) and (2) in (3), we obtain at once

(18) $$\tan(\theta_j) = \frac{U' \cdot t_j - x_j}{V' + W' \cdot t_j - y_j} \quad ; \quad j = 1,..,n$$

where

$$U' \stackrel{df}{=} V_x$$

$$V' \stackrel{df}{=} r_{y_o}$$

$$W' \stackrel{df}{=} V_y$$

Clearly, in any physically realizable situation (U', V', W') corresponds to an actual target and may take on any set of possible values (within reason) and generate the "observations' θ_j , $j = 1,..,n$.

Thus, we can assume (18) has a solution for U', V', W', for the given set of θ_j 's.

Suppose, (U'', V'', W'') is some other solution to (18), and first suppose $U'' \neq U'$. Thus

(19) $$\tan(\theta_j) = \frac{U'' \cdot t_j - x_j}{V'' + W'' t_j - y_j} \quad ; \quad j = 1,.., n \quad .$$

Multiplying (18) through by its denominator and (19) by its denominator, substracting and then resolving for $\tan(\theta_j)$, yields

(20) $$\tan(\theta_j) = \frac{(U' - U'') \cdot t_j}{(V' - V'') + (W' - W'') \cdot t_j}$$

$$= \frac{t_j}{\alpha + \beta \cdot t_j} \quad ;$$

$$j = 1,.. , n \quad ,$$

where $\alpha \stackrel{df}{=} \frac{V' - V''}{U' - U''}$, $\beta \stackrel{df}{=} \frac{W' - W''}{U' - U''}$.

But, the same set of θ_j's occurs in (18), and hence, eliminating $\tan\theta_j$ from (18) and (20) yields

$$(21) \qquad \frac{t_j}{\mathcal{Q} + \mathcal{B} \cdot t_j} = \frac{U' \cdot t_j - x_j}{V' + W' \cdot t_j - y_j} \; ; \; j = 1,..., n.$$

Equivalently,

$$(22) \qquad (\mathcal{Q} + \mathcal{B} \cdot t_j) \cdot (U' t_j - x_j) = t_j \cdot (V' + W' t_j - y_j)$$

for $j = 1, 2, .., n$.

Collecting coefficients of the powers of the t_j's in the above identify, using (17), yields:

$$(23) \qquad (U' - \alpha_1) \cdot \mathcal{Q} \cdot t_j + (-\alpha_2 \cdot \mathcal{Q} + (U' - \alpha_1) \cdot \mathcal{B}) \cdot t_j^2 + (-\alpha_3 \cdot \mathcal{Q} - \alpha_2 \cdot \mathcal{B}) \cdot t_j^3 - a \cdot 0_x (t_j^4) \cdot \mathcal{Q} - t_j (\alpha_3 t_j^3 + a \cdot 0_x (t_j^4)) \cdot \mathcal{B}$$

$$= V' \cdot t_j + (W' - \beta_1) \cdot t_j^2 - \beta_2 t_j^3 - b \cdot t_j \; 0_y (t_j^3),$$

for $j = 1, 2, ..., n$.

Equating coefficients of the powers of the t_j's in the identity in (23) gives the following overdetermined system in $\mathcal{Q}$ and $\mathcal{B}$.

$$(24) \quad (a) \qquad (U' - \alpha_1) \cdot \mathcal{Q} = V'$$

$$(b) \qquad -\alpha_2 \cdot \mathcal{Q} + (U' - \alpha_1) \cdot \mathcal{B} = W' - \beta_1$$

$$(c) \qquad \alpha_3 \cdot \mathcal{Q} + \alpha_2 \cdot \mathcal{B} = \beta_2,$$

and, in addition, $\mathcal{Q}$ and $\mathcal{B}$ must satisfy the relation

$$(25) \qquad a \cdot 0_x (t_j^4) \cdot \mathcal{Q} + t_j \cdot (\alpha_3 t_j^3 + a \cdot 0_x (t_j^4)) \cdot \mathcal{B}$$

$$= b \cdot t_j \cdot 0_y (t_j^3), \text{ for } j = 1, .., n.$$

We now also suppose (temporarily) that $U' \neq \alpha_1$. Then solving in (24a) for $\mathcal{Q}$ and then in (24b) for $\mathcal{B}$ and finally substituting into (24c) we obtain

(26)(a) $$\mathcal{Q} = \frac{V'}{U' - \alpha_1},$$

(b) $$\mathcal{B} = \frac{W' - \beta_1}{U' - \alpha_1} + \frac{\alpha_2 \cdot V'}{(U' - \alpha_1)^2},$$

(c) $$\frac{\alpha_3 \cdot V'}{U' - \alpha_1} + \frac{\alpha_2 \cdot (W' - \beta_1)}{U' - \alpha_1} + \frac{\alpha_2^2 \cdot V'}{(U' - \alpha_1)^2} = \beta_2$$

Simplifying (26) (c), we can solve for V' :

(27) $$V' = \frac{(\beta_2 \cdot (U' - \alpha_1) - \alpha_2 \cdot (W' - \beta_1)) \cdot (U' - \alpha_1)}{\alpha_3 \cdot (U' - \alpha_1) + \alpha_2^2},$$

W', U' arbitrary.

But, in general, (except with probability zero, given all values equal) V' need not be related to U', W' and as well $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2$. Indeed, any possible triple of values of (U', V', W') can occur, which in turn with the α_j's and β_j's - also chosen separately - generate the 'observations' θ_j's.

Equation (27) and its consequential linear restricting relations for U', V', W' for non-uniqueness to hold in (18) remain valid even when $\alpha_3 = 0$.

If $U' = \alpha_1$, then (18) becomes

(28) $$\begin{pmatrix} y_1 - 0_x\,(t_1^2) \cdot \cot(\theta_1) \\ \vdots \\ y_n - 0_x\,(t_n^2) \cdot \cot(\theta_n) \end{pmatrix} = \begin{pmatrix} 1 & t_1 \\ \vdots & \vdots \\ 1 & t_n \end{pmatrix} \cdot \begin{pmatrix} V' \\ W' \end{pmatrix},$$

an ordinary linear regression on V' and W', implying, analagous to (6), a unique solution for V' and W'.

Now suppose U' = U''. This implies that if (U'', V'', W'') is any other solution,

(29)
$$\tan(\theta_j) = \frac{U'\cdot t_j - x_j}{V' + W'\cdot t_j - y_j}$$

$$= \frac{U'\cdot t_j - x_j}{V'' + W''\cdot t_j - y_j},$$

$$j = 1, 2, \ldots, n ,$$

and hence

(30)
$$V' + W'\cdot t_j - y_j = V'' + W''\cdot t_j - y_j ,$$

$$\text{for} \quad j = 1, 2, \ldots, n .$$

This implies by equating coefficients, immediately that V' = V'' and W' = W''.

Thus, in general, for all cases, (U'', V'', W'') = (U', V', W'), and equation (18) has a unique solution.

Hence, when we purposely make the monitoring aircraft motion nonlinear, in general a unique solution exists in equation (18) for that target generating the path.

The case where error is present is analagous and we can thus apply nonlinear regression or linearized regression techniques (direct filtering or Kalman filtering) to the model.

$$\text{(31)} \qquad \tan(\hat{\theta}_j) = \frac{U' \cdot t_j - x_j}{V' + W' \cdot t_j - y_j} + \epsilon_j ,$$

$$j = 1,.., n$$

to obtain, in general, unique estimates of U', V', W' (at least approximately optimal).

(For examples of these techniques, see references 2 and 3.)

The higher the degree m of the monitoring aircraft equations of motion, the more restrictive the 'zero-probability' conditions on U', V', W, because (from equating coefficients of the higher powers of t_j) of the additional equations

$$\text{(32)} \qquad \left\{ \begin{array}{lll} \alpha_3 \cdot \mathcal{Q} & + \alpha_2 \cdot \mathcal{B} & = \beta_2 \\ \vdots & \vdots & \vdots \\ \alpha_m \cdot \mathcal{Q} & + \alpha_{m-1} \cdot \mathcal{B} & = \beta_{m-1} \\ & \alpha_m \cdot \mathcal{B} & = \beta_m \end{array} \right.$$

to (24) (a) and (b).

Thus, for example, if m = 3 and hence $\alpha_3 \neq 0$, it follows that

$$\text{(33)} \qquad \frac{V'}{U' - \alpha_1} = \frac{\beta_2}{\alpha_3} - \frac{\alpha_2 \cdot \beta_3}{\alpha_3^2}$$

$$\frac{W' - \beta_1}{U' - \alpha_1} + \frac{\alpha_2 \cdot V'}{(U' - \alpha_1)^2} = \frac{\beta_3}{\alpha_3}$$

Equivalently

(34)
$$\begin{cases} V' = \left(\dfrac{\beta_2}{\alpha_3} - \dfrac{\alpha_2 \cdot \beta_3}{\alpha_3{}^3}\right) \cdot (U' - \alpha_1) \\[2ex] W' = \beta_1 - \alpha_2 \cdot \left(\dfrac{\beta_2}{\alpha_3} - \dfrac{\alpha_2 \cdot \beta_3}{\alpha_3{}^2}\right) + \dfrac{\beta_3}{\alpha_3} \cdot (U' - \alpha_1), \\[2ex] U' \text{ arbitrary} \end{cases}$$

If $m \geq 4$, then either (32) is a consistent linear system of equations in $\mathcal{Q}$ and $\mathcal{B}$ or inconsistent.

If the α_i's and β_i's are chosen such that

(35)
$$\begin{pmatrix} \beta_2 \\ \vdots \\ \beta_{m-1} \\ \beta_m \end{pmatrix} = \lambda_1 \cdot \begin{pmatrix} \alpha_3 \\ \vdots \\ \alpha_m \\ 0 \end{pmatrix} + \lambda_2 \cdot \begin{pmatrix} \alpha_2 \\ \vdots \\ \alpha_{m-1} \\ \alpha_m \end{pmatrix}$$

for some real λ_1, λ_2, then (32) is consistent, and thus (34) is the extent to which U', V', W' can be restricted for nonuniqueness.

On the other hand, if the α_i's and β_i's are chosen such that (35) does not hold, then (32) is inconsistent and no $\mathcal{Q}$ and $\mathcal{B}$ can satisfy (32), and hence we have a complete contradiction to the assumption of nonunnique-ness of solutions of (18).

In particular, if the monitoring aircraft flies a constant tangential velocity circular path with angular speed ω beginning at the origin at $t_o = 0$, the circle having a center (see Figure 2) located on the positive y - axis at $\binom{0}{r}$, then eq. (17) becomes

(36)
$$\begin{cases} x_j = r \cdot \sin(\omega \cdot t_j) \\ y_j = r \cdot (1 - \cos(\omega \cdot t_j)); \quad j = 1,2,..,n \end{cases}$$

Since the sine and cosine are entire functions we have the usual Taylor series expansions valid for <u>all</u> t , j =1..,n,

(37)
$$\begin{cases} x_j = \sum_{i=1}^{+\infty} \alpha_i \cdot t_j^{\,i} \\ \\ y_j = \sum_{i=1}^{+\infty} \beta_i \cdot t_j^{\,i} \end{cases},$$

where
$$\alpha_{2i-1} = r\cdot(-1)^{i+1}\cdot\frac{\omega^{2i-1}}{(2i-1)!}$$
$$\alpha_{2i} = 0$$
$$\beta_{2i} = r\cdot(-1)^{i+1}\cdot\frac{\omega^{2i}}{(2i)!}$$
$$\beta_{2i-1} = 0 ,$$

i = 1, 2, 3,.. .

All the analysis in eq.'s (17) - (35) is valid here with the degree of the path being $m = +\infty$. Thus (32) becomes

(38)
$$\begin{cases} \alpha_3\cdot\mathcal{Q} = \beta_2 \\ \alpha_3\cdot\mathcal{B} = 0 \\ \alpha_5\cdot\mathcal{Q} = \beta_4 \\ \alpha_5\cdot\mathcal{B} = 0 \\ \cdots\cdots\cdots\cdots \end{cases}$$

But this implies

(39)
$$\begin{cases} \mathcal{B} = 0 \\ \mathcal{Q} = \dfrac{\beta_2}{\alpha_3} = \dfrac{\beta_4}{\alpha_5} = \cdots \end{cases} \text{and}$$

Now, we remark, for example, that

(40) $$\frac{\beta_2}{\alpha_3} = \frac{r \cdot \omega^2/2!}{-r \cdot \omega^3/3!}$$

$$= -3 \cdot \omega$$

while

(41) $$\frac{\beta_4}{\alpha_5} = \frac{-r \cdot \omega^4/4!}{r \cdot \omega^5/5!}$$

$$= -5 \cdot \omega \, .$$

Hence we have a contradiction in (39) and thus (32) is an inconsistent system for the circular motion here.

Thus, no $\mathcal{Q}$ and $\mathcal{B}$ can satisfy (32) and consequently eq. (18) has a unique solution here.

Also, we comment that if the monitoring aircraft motion is only piecewise described by a power series in time, similar uniqueness results must hold for the solution of (18), by modifying the previous arguments beginning in eq. (21).

Lastly, we remark that much care must be exercised for the type of nonlinear motion determined for x_j and y_j in equation (31) for the presence of ill-conditioning in the appropriate approximation algorithm used.

SUMMARY OF ANALYSIS

Target always describes straight line constant velocity motion.

1. LINEAR MONITORING AIRCRAFT MOTION

If all target parameters are unknown a priori, then whether bearing

observation error is present or not, the target is not uniquely determinable or uniquely estimatable from the data. (See (9) for the set of all ambiguous parameter values for the zero error case and (14) for the general error case.) If one of the target parameters is known, a priori, then the target motion is uniquely determined. (For x-velocity, V_x, of the target known, see (15) and (16).)

2. NONLINEAR MONITORING AIRCRAFT MOTION

If all target parameters are unknown a priori, then for both the zero error and general error cases for bearing observations, the target is essentially uniquely determinable (or estimable). The possible exceptions - 'zero probability situations, given all things equally possible' - are linear restrictions on the unknown parameters. (See (27) for second degree motion and the more restrictive (34) for third degree motion.)

When the monitoring aircraft motion is of sufficiently high degree ($m \geq 4$), in general, there are not even any 'zero probability' linear restrictions on the unknown parameters which can produce a nonunique solution to the bearing observation equations ((18)). (See eq. (35) and following remarks.)

For the case of circular aircraft motion (see (36) - (41)), the above remark is valid, i.e., the target motion is uniquely determined from the bearing observations, with no exceptions.

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